Math 4

Unit 4: Trig Identities - Proofs/Solving

"C" Problems

① $\cos^3 x + \sin^2 x \cdot \cos x = \cos x$

$\cos x(\cos^2 x + \sin^2 x) =$

$\cos x \cdot 1 =$

$\cos x = \cos x$

② $\sec x \sin x = \tan x$

$\frac{1}{\cos x} \cdot \sin x =$

$\tan x = \tan x$

③ $\frac{\sec^2 x - 1}{\tan x} = \tan x$

$\frac{\tan^2 x}{\tan x}$

$\tan x = \tan x$

④ $\sec x \cdot \sin x \cdot \cot x = 1$

$\frac{1}{\cos x} \cdot \frac{\sin x}{1} \cdot \frac{\cos x}{\sin x} =$

$1 = 1$

⑤ $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$1 - \sin^2 x - \sin^2 x =$

$1 - 2\sin^2 x = 1 - 2\sin^2 x$

⑥ $\frac{1 + \tan x}{\tan x} = 1 + \cot x$

$\frac{1}{\tan x} + \frac{\tan x}{\tan x} =$

$\cot x + 1 = 1 + \cot x$

⑦ $\frac{\cos x + \tan x}{\sin x} = \sec x + \cot x$

$\frac{\cos x}{\sin x} + \frac{\frac{\sin x}{\cos x}}{\sin x} =$

$\cot x + \frac{1}{\cos x} =$

$\cot x + \sec x = \sec x + \cot x$

⑧ $1 - \sin x \cdot \cos x \cdot \tan x = \cos^2 x$

$1 - \sin x \cdot \cos x \cdot \frac{\sin x}{\cos x} =$

$1 - \sin^2 x =$

$\cos^2 x = \cos^2 x$

(9) $1-2\sin^2 x = 2\cos^2 x - 1$

$1-2(1-\cos^2 x) =$

$1-2+2\cos^2 x =$

$2\cos^2 x - 1 = 2\cos^2 x - 1$

(10) $\dfrac{\sin x \cdot \cot x + \cos x}{\sin x} = 2\cot x$

$\dfrac{\sin x \cdot \cot x}{\sin x} + \dfrac{\cos x}{\sin x} =$

$\cot x + \cot x =$

$2\cot x = 2\cot x$

(11) $\cos x = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$x = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

(12) $\sec x = 2$

$\cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

(13) $2\sin x = 6$

$\sin x = 3$

No solution

(14) $\sin x = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

(15) $\tan x = \dfrac{1}{\sqrt{3}}$

$x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

"B" Problems

(16) $\dfrac{\tan x}{1+\tan^2 x} = \sin x \cdot \cos x$

$$\frac{\frac{\sin x}{\cos x}}{\sec^2 x} =$$

$$\frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{1} =$$

$$\sin x \cdot \cos x = \sin x \cdot \cos x$$

(18) $\dfrac{\cos x}{1-\sin x} = \dfrac{1+\sin x}{\cos x}$

$$\left(\frac{1+\sin x}{1+\sin x}\right)\left(\frac{\cos x}{1-\sin x}\right) =$$

$$\frac{\cos x(1+\sin x)}{1-\sin^2 x} =$$

$$\frac{\cos x(1+\sin x)}{\cos^2 x} =$$

$$\frac{1+\sin x}{\cos x} = \frac{1+\sin x}{\cos x}$$

(17) $\sin x \cdot \tan x + \cos x = \sec x$

$$\sin x \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{1} =$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} =$$

$$\frac{1}{\cos x} =$$

$$\sec x = \sec x$$

(19) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$$\cos^2 x - (1-\cos^2 x) =$$

$$\cos^2 x - 1 + \cos^2 x =$$

$$2\cos^2 x - 1 = 2\cos^2 x - 1$$

(20) $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$

$$\frac{(1+\sin x)-(1-\sin x)}{1-\sin^2 x} =$$

$$\frac{2\sin x}{\cos^2 x} =$$

$$\frac{2}{1} \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} =$$

$$2\tan x \cdot \sec x = 2\tan x \sec x$$

(21) $\sin x + \sin x \cdot \cot x = 0$

$\sin x(1 + \cot x) = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$\cot x = -1$

$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

(22) $2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x = 1$

$x = 0, 2\pi$

(23) $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(24) $\tan^2 x \cdot \sin x = -\sin x$

$\tan^2 x \cdot \sin x + \sin x = 0$

$\sin x(\tan^2 x + 1) = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$\tan^2 x + 1 = 0$

$\tan^2 x = -1$

No solution

(25) $\csc x = \sin^2 x \cdot \csc x$

$0 = \sin^2 x \cdot \csc x - \csc x$

$\csc x(\sin^2 x - 1) = 0$

$\csc x = 0$

$\frac{1}{\sin x} = 0$

No solution

$\sin^2 x - 1 = 0$

$\sin^2 x = 1$

$\sin x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

(26) $2\sin\left(x + \frac{\pi}{6}\right) = 1$

$\sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$

$x + \frac{\pi}{6} = \frac{\pi}{6}$

$x = 0$

$x + \frac{\pi}{6} = \frac{5\pi}{6}$

$x = \frac{4\pi}{6}$

$x = \frac{2\pi}{3}$

"A" Problems

(27) $(1+\csc x)(1-\sin x) = \cot x \cdot \cos x$

$1 + \csc x - \sin x - \csc x \cdot \sin x =$

$1 + \csc x - \sin x - 1 =$

$\frac{1}{\sin x} - \sin x \left(\frac{\sin x}{\sin x}\right) =$

$\frac{1 - \sin^2 x}{\sin x} =$

$\frac{\cos^2 x}{\sin x} =$

$\frac{\cos x}{\sin x} \cdot \cos x =$

$\cot x \cdot \cos x = \cot x \cdot \cos x$

(28) $\frac{1 + \sec x}{\tan x + \sin x} =$

$\left(\frac{\cos x}{\cos x}\right) \frac{1 + \frac{1}{\cos x}}{\frac{\sin x}{\cos x} + \sin x} =$

$\frac{\cos x + 1}{\sin x + \sin x \cos x} =$

$\frac{\cos x + 1}{\sin x (1 + \cos x)} =$

$\frac{1}{\sin x} =$

$\csc x = \csc x$

(29) $\dfrac{1}{\cot x - \tan x} = \dfrac{\sin x \cdot \cos x}{1 - 2\sin^2 x}$

$\left(\dfrac{\sin x \cdot \cos x}{\sin x \cdot \cos x}\right) \dfrac{1}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} =$

$\dfrac{\sin x \cdot \cos x}{\cos^2 x - \sin^2 x} =$

$\dfrac{\sin x \cdot \cos x}{1 - \sin^2 x - \sin^2 x} =$

$\dfrac{\sin x \cdot \cos x}{1 - 2\sin^2 x} = \dfrac{\sin x \cdot \cos x}{1 - 2\sin^2 x}$

(30) $\sec^2 x \cdot \sin^2 x + (\sin x + \cos x)^2 - \sec^2 x = 2 \sin x \cdot \cos x$

$\frac{1}{\cos^2 x} \cdot \sin^2 x + \underline{\sin^2 x} + 2\sin x \cdot \cos x + \underline{\cos^2 x} - \sec^2 x =$

$\underline{\tan^2 x} + \underline{1} + 2\sin x \cdot \cos x - \underline{\sec^2 x}$

$\underline{-1} + 1 + 2\sin x \cos x =$

$2\sin x \cdot \cos x = 2\sin x \cdot \cos x$

(31) $\dfrac{1+\sec x}{\tan x+\sin x} = \csc x$

$\dfrac{1+\frac{1}{\cos x}}{\frac{\sin x}{\cos x}+\sin x}\left(\dfrac{\cos x}{\cos x}\right) =$

$\dfrac{\cos x+1}{\sin x+\sin x\cdot\cos x} =$

$\dfrac{\cos x+1}{\sin x(1+\cos x)} =$

$\dfrac{1}{\sin x} =$

$\csc x = \csc x$

(32) $\dfrac{\cos^3 x+\sin^3 x}{\cos x-\cos^2 x\cdot\sin x} = 1+\tan x$

$\dfrac{(\cos x+\sin x)(\cos^2 x-\cos x\cdot\sin x+\sin^2 x)}{\cos x(1-\cos x\cdot\sin x)} =$

$\dfrac{(\cos x+\sin x)(1-\cos x\cdot\sin x)}{\cos x(1-\cos x\cdot\sin x)} =$

$\dfrac{\cos x}{\cos x}+\dfrac{\sin x}{\cos x}$

$1+\tan x = 1+\tan x$

(33) $\csc^4 x-\cot^4 x = 2\csc^2 x-1$

$(\csc^2 x+\cot^2 x)(\csc^2 x-\cot^2 x) =$

$(\csc^2 x+\cot^2 x)(1) =$

$\csc^2 x+\csc^2 x-1 =$

$2\csc^2 x-1 = 2\csc^2 x-1$

(34) Recall: $\sin^2 x+\cos^2 x = 1$

$\dfrac{1+2\sin x\cdot\cos x}{\sin x+\cos x} = \sin x+\cos x$

$\dfrac{\sin^2 x+\cos^2 x+2\sin x\cdot\cos x}{\sin x+\cos x} =$

$\dfrac{\sin^2 x+2\sin x\cdot\cos x+\cos^2 x}{\sin x+\cos x} =$

$\dfrac{(\sin x+\cos x)^2}{\sin x+\cos x} =$

$\sin x+\cos x = \sin x+\cos x$

(35) $\dfrac{\cos^2 x + 3\sin x - 1}{\cos^2 x + 2\sin x + 2} = \dfrac{1}{1+\csc x}$

$\dfrac{1-\sin^2 x + 3\sin x - 1}{1-\sin^2 x + 2\sin x + 2} =$

$\dfrac{-\sin^2 x + 3\sin x}{-\sin^2 x + 2\sin x + 3} =$

$\dfrac{\sin x(-\sin x + 3)}{(\sin x + 1)(-\sin x + 3)} =$

$\left(\dfrac{\csc x}{\csc x}\right)\dfrac{\sin x}{\sin x + 1} =$

$\dfrac{1}{1+\csc x} = \dfrac{1}{1+\csc x}$

(36) $\dfrac{1}{\sec x - \tan x} = \sec x + \tan x$

$\dfrac{\sec^2 x - \tan^2 x}{\sec x - \tan x} =$

$\dfrac{(\sec x + \tan x)(\sec x - \tan x)}{\sec x - \tan x}$

$\sec x + \tan x = \sec x + \tan x$

(37) $-2\sin^2 x - 5\cos x + 4 = 0$

$-2(1-\cos^2 x) - 5\cos x + 4 = 0$

$-2 + 2\cos^2 x - 5\cos x + 4 = 0$

$2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$2\cos x - 1 = 0$ | $\cos x - 2 = 0$

$\cos x = \frac{1}{2}$ | $\cos x = 2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ | No solution

(38) $2\cos^2 x + \sin x - 1 = 0$

$2(1-\sin^2 x) + \sin x - 1 = 0$

$2 - 2\sin^2 x + \sin x - 1 = 0$

Magic $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0$ | $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ | $\sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ | $x = \frac{\pi}{2}$

(39) $\tan^2 x - \sec x - 1 = 0$

$\sec^2 x - 1 - \sec x - 1 = 0$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0$ | $\sec x + 1 = 0$

$\sec x = 2$ | $\sec x = -1$

$\cos x = \frac{1}{2}$ | $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ | $x = \pi$